
UNIT 2 AIMS AND OBJECTIVES OF TEACHING-LEARNING MATHEMATICS

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INTRODUCTION

Whatever may be the subject, any teaching-learning is organised keeping in view the aims that are broad and comprehensive in nature. The aims are achievable through the realization of objectives that are narrow and specific. Teaching-learning of mathematics also takes account of well defined objectives. Developing mathematical knowledge, understanding various mathematical terms, widening appreciation towards Mathematics, and expanding skill of applying acquired knowledge in everyday activities are some among them. This Unit explains the most critical objectives expected to be achieved by teaching-learning of Mathematics namely; mathematisation of children's mind, development of reasoning power and visualisation, expansion of the skill of problem solving, increasing the power of critical thinking and so on. Also, Mathematics and its relation with life and its integration with other subjects are also discussed.

OBJECTIVES

After completing this unit, you will be able to:

- identify the need for establishing aims and objectives of teaching mathematics;
- state aims and objectives of teaching mathematics upper primary and secondary level;

- understand meaning of mathematisation and ways of mathematisation of mind ,
- distinguish between inductive and deductive reasoning;
- explain the use of visualisation in validating mathematical knowledge;
- describe steps involved in problem solving;
- comprehend the skills that help in solving mathematical problems;
- suggest activities to develop critical thinking in Mathematics;
- explain relationship between Mathematics and experiences in life; and
- discuss the relationships between Mathematics with other subjects.

AIMS AND OBJECTIVES OF TEACHING MATHEMATICS

Teachers need to well aware about the different aims and objectives of Mathematics teaching. A teacher who has sufficient knowledge on these aspects would be able to develop various skills among his/her students. Let us discuss the aims and objectives of teaching Mathematics.

Aims of Teaching Mathematics

First of all let us try to understand the difference between goal, aim and objective of teaching Mathematics. The term 'aims of teaching Mathematics', stands for the goal or broad purpose that needs to be fulfilled by the teaching of that subject in the general scheme of education. Goals and Aims are like ideals, and their attainment needs long term planning. Therefore, they are divided into some definite and workable units named as objectives. The specific objectives are those short term, immediate goals and purpose that may be achieved within the specified classroom transactions" (NCERT, 2012). Thus we can conclude that aim is more broad, comprehensive and general in nature while objectives are means to achieve the aim.

According to NCGF-2005, the main goal of Mathematics education in school is the mathematization of minds of children.

What are the aims of teaching Mathematics? A brief description of the general aims of teaching Mathematics is given in Figure 2.1

• Mathematics has numerous applications in one's daily life; may be for/rit g shopping, planning different activities regarding time calculations, etc. There are inclined to utilize the various concepts of mathematics in multiple situations to accomplish the tasks. This exhibits the utilitarian aims of mathematics.

• Mathematics being a discipline that aims to develop the characteristics such as accuracy, reasoning, problem solving skills, thoroughness, patience, etc.

• The development of literacy and social civilization has close relationship with mathematics. Mathematics has considerably contributed in the developmental stages of various disciplines like science, engineering, medicine, art, music etc. Those cultural allies of mathematics need to be preserved and attained through its teaching

• Mathematics education helps individuals to emerge as skilled persons and to grow independent to earn their livelihood. Thus the vocational aim of mathematics pertains to help individuals to take up specialized vocations having application of mathematics i.e. engineering, accounting, banking, etc

• Mathematics and society are closely related. Mathematics is being extensively employed in planning developmental programmes of society. For example, while constructing roads and bridges, its concepts like 'measurement', 'time', etc are used

• As children engage in mathematics learning (say for example, problem solving situation) different moral values are developed in them. For example, skills like patience, self effort, etc grow among children

Fig. 2.1: General Aims of Teaching Mathematics

Objectives of Teaching Mathematics

We understand that aims are ideal general statements that are broad and comprehensive. Also aims are not definite and clear and require long term planning to achieve. In such a case, we move on to a more clear, achievable and workable units called objectives. Objectives are definite, clear, narrow, specific and can be attained in a short duration.

There are two types of objectives of teaching: general and specific. General educational objectives are broad and related to school and educational system.

Following are the general objectives of teaching Mathematics at secondary level.

The students will be able to:

- Acquire knowledge of facts, concepts, theories, laws, principles, proofs of Mathematics;
- Develop the ability to communicate mathematical ideas with precision and accuracy;

- Develop interest and positive attitude towards Mathematics;
- Apply mathematical knowledge to solve real life problems;
- Develop the skill to use algorithms in problems solving;
- Appreciate the contributions of mathematicians;
- Develop mastery of algebraic skills, drawing skills, deducing interpretations, finding patterns, making connections, analyse, organise data, reasoning, critical thinking, etc.

Specific objectives are short term immediate goals or purposes that may be achieved through classroom instructional/educational process. Thus we may call such a objective as **educational objective/instructional objective**. In the case of classroom instructions, teacher is concerned about bringing changes in the behaviour of learners and we call that specific objective as **behavioural objective**. Behavioural objectives are the objectives that written in behavioural terms. It explains the change in state of behaviour of an individual on completion of a learning activity.

As we discussed earlier, educational processes aspire for bringing behavioural changes in an individual. Bloom has classified the change in behaviour in three domains or categories namely; cognitive domain, affective domain and psychomotor domain. Bloom (1956) had organised various educational objectives in a hierarchical order and we call it as **Bloom's Taxonomy of Educational Objectives**. The educational objectives falling in each domain is hierarchically placed in ascending order of complexity. Even though, many taxonomies are available the most acceptable and widely used one is Bloom (revised) and others for cognitive domain; by Krathwohl for affective domain and Dave for psychomotor domain.

Cognitive Domain Bloom(1956)	Affective Domain Krathwohl, Bloom, Masia (1973)	Psychomotor Domain Dave(1975)
Knowledge	Receiving	Imitation
Comprehension	Responding	Manipulation
Application	Valuing	Precision
Analysis	Organization	Articulation
Synthesis	Internalizing Values (Characterization)	Naturalization
Evaluation		

Table 2.1: Taxonomy of Educational Objectives

In the year 2001, Anderson and Krathwohl modified the objectives belonging to the cognitive domain of Bloom's taxonomy. The pictorial representation of Anderson and Krathwohl Taxonomy 2001 is given in Figure 2.2.

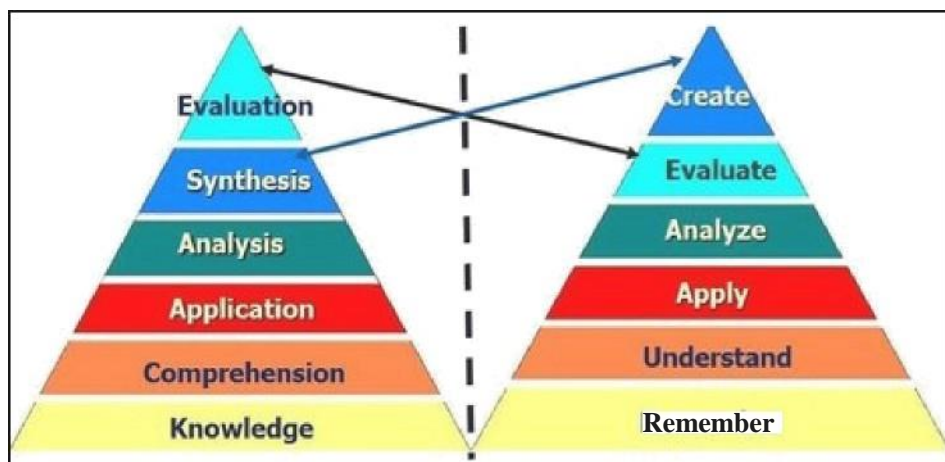


Fig. 2.2: Anderson and Krathwohl Taxonomy 2001

(Source: <http://www.scoop.it/t/cognitive-rigor?page=2>)

Keeping in view the context of objectives of teaching Mathematics let us briefly discuss the taxonomy of educational objectives. It is the duty of the teacher to first locate different learning (specific) objectives pertaining to the topic/concept that he/she is going to teach. After deciding the learning objective, the different learning experiences/activities and assessment mechanisms are chosen. Thus the learning objectives are written with the help of action verbs that are clear and specific. The action verbs give the direction to the teacher about 'what the children will do' or 'what the learners are expected to do' after completion of the learning activity. Thus, with each learning objective, action verbs and specifications are associated. Now let us discuss the learning objectives belonging to different domains.

Cognitive Domain: Cognitive means 'knowledge'. The levels falling under cognitive domain are as follows:

Remember: 'Remembering' is the lowest level objective of cognitive domain. It refers to the ability to recall information of facts, concepts, theories, laws, patterns, structures, generalizations, etc. A child who has the ability to recall mathematical information would be able to proceed to acquire highest learning.

Examples: recalling the definition of rectangle, formula for finding the area of rectangle.

Understand: It is the next higher level of cognitive domain. Understanding helps learners to correlate, connect and develop meaning from new material.

Example: describing the method of calculating area of rectangle.

Apply: The learner is able to apply different facts, concepts, theories, laws or principles in new situation. Application subsumes both knowledge and understanding.

Example: applying knowledge of calculating area of rectangle to find the area of own house of the learner.

Analyze: Analysis is the breaking down of a complex situation into different parts/elements. At such a stage, the learner will be able to locate the elements, differentiate, recognize relationship, and identify patterns pertaining to a situation.

Example: identify the causes of splitting the following figure into different parts for calculating its total area.

Evaluate: Evaluation represents the learner's ability to formulate hypothesis, critique, and judge a material, situation or method against the standard, which may be internal or external to it. Evaluation is the most complex mental process belonging to cognitive domain.

Example: justifying the need for constructing 'rooms' in rectangular/square shapes.

Create: As the word implies creating stands for collecting information, designing and putting elements together to construct a new pattern or develop theory out of it or to build up understanding of the concept in detail.

Example: making a rectangular shaped house using thermocol sheet.

Affective Domain: It deals with the emotional aspects of the learner. The various emotional states of an individual like different feelings, motivation, interest, attitude, values, appreciation, etc fall under affective domain. Similar to cognitive domain, the learning objectives of affective domain are hierarchically ordered i.e. from simple to complex. The learning objectives pertaining to affective domain are given below:

Receiving: Receiving is the lowest level objective of affective domain. Receiving refers to the learner's ability to listen and receive a situation, stimuli, phenomenon, information, etc. For example, listening to the teacher's lecture on the topic 'area of rectangle'.

Responding: In this stage of mental process, the learner starts responding to different situations, information and stimuli. For example, asking teacher the difference between perimeter and area of the rectangle.

Valuing: Valuing involves increasing internalisation of the worth or value a person attached to a particular object, phenomenon or behaviour (NCERT, 2012). For example, showing interest in solving problems related to rectangle.

Organisation: Organisation is the fourth level objective which brings together different values, resolving conflicts between them, starting to build an internally consistent and a unique value system or attitude (NCERT, 2012). For example, showing the attitude to solve mathematical problems by self.

Internalising Values (Characterisation): Characterisation is the highest level objective in which the values and attitudes of an individual is attained to help to control his/her behaviour. The personal, social and emotional behaviour of an individual reflects his/her attainment of values. For example, while solving mathematical problems, maintaining patience till he/she reaches answer.

Psychomotor Domain: Psychomotor domain includes the ability to use body parts to accomplish tasks, neuro muscular movements and types of body actions. The psychomotor skills can be observed while playing, typing, stitching, etc. Psychomotor skills are developed through practice and are measured in terms of speed, precision, accuracy etc. The objectives belonging to psychomotor domain are given below:

Imitation: Imitation is the lowest level objective of psychomotor domain. At this stage, individual observes actions and are practiced/repeated/simulated at his/her mental level. Later, the individual performs those actions but with less precision. For example, constructing a rectangle by using matchsticks.

Manipulation: Manipulation involves listening to other's directions, selecting certain actions in preference to others and practicing those actions for accuracy

and perfection. For example, listening to the teacher and build a rectangle as per the teacher's advice.

Precision: The third level objective of psychomotor domain implies the development of motor skills with exactness. At this stage, the control over actions helps him/her to develop required motor skills with precision. For example, assembling various objects, take measurements and then construct a rectangle with accurate measurements.

Articulation: Articulation involves control over multiple motor skills in a logical and systematic way which help individual to complete the desired action. High level coordination of various motor skills is developed at this stage. For example, to construct a rectangle, firstly, the items needed are collected, measurements are taken, the procedures are followed and jotted down then finally the rectangle is made in a sequential order.

Naturalistaion: The highest level of psychomotor domain, at this level, motor skills and coordination of movements becomes the reflex actions/mechanical. While performing any action, the individual naturally performs with precision and accuracy.

For example, when children are asked to construct a rectangle, automatically the materials required comes to their mind, and they succeeds in constructing it.

Formulation of objective will guide you in your teaching learning process and in turn help children to achieve the desired learning outcomes.

Now we may discuss few of the general aim and objectives in detail like mathematisation of mind of children, enhancement of reasoning power and visualization, developing problem solving skills and critical thinking in Mathematics.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

1) What are the general aims of teaching mathematics? Write any three aims.

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2) What are the general objectives of teaching Mathematics at secondary level? Write any three objectives.

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3) Write any four specific objectives for the unit 'Probability'.

MATHEMATISATION OF MINDS OF CHILDREN

Mathematics is a subject that has relevance to day-to-day life. For example, a child buys a notebook costing T 30 and gives the shopkeeper a T 50 note. Here, the shopkeeper returns T20 to the child. What mathematical thinking is involved here? The shopkeeper has used the basic mathematical operation 'subtraction', while returning N0 to the child. This way, each individual utilizes Mathematics in his/her day-to-day life. How is it possible for an individual to think in mathematical terms? Teachers have great role in this regard. As we know, apart from teaching mathematical facts, teachers should help children to develop the skill of utilizing Mathematics and seeing things in mathematical way. Mathematics is not for acquiring good grades, rather, it is for helping children to be able to deduce mathematical interpretations of the circumstances and judge through mathematical thinking. ***Mathematisation refers to the act of interpreting or expressing mathematically, or the state of being considered or explained mathematically.*** In other words mathematisation is nothing but converting a given situation in a more specific way using numerals, symbols and mathematical facts satisfying the given conditions with the help of logic. Logic is the back bone towards mathematisation of any given situation.

The teaching- learning of mathematics stress much on developing the skill of mathematisation. It is expected that, children should expand the horizon of cognition by incorporating abilities that help them manage situations mathematically. Here, children should make use of the knowledge, facts and principles of Mathematics to arrive at a judgement. This is referred to as mathematisation of mind. A child, who has developed the ability to think in mathematical way, is capable of tackling and finding solutions to all problems that she/he encounters. So, it is a must on the part of teachers that they should organise learning activities that emphasise the skill of development of mathematisation. Let us look at how Mr. Jagat has organised an activity to achieve this aim.

Activity 1

Mr. Jagat is teaching his seventh class students. He divides the class into different groups. He told, “Today we are going to play a game. It is a simple game. Two students will act as shop keepers. They will sell materials such as notebooks, pens, lunch boxes, etc. The rest of the students should buy materials from them. You should buy materials for yourself and your brothers/sisters. Finally, you should prepare a chart containing details of all materials that you have purchased. Also, you should write one more thing i.e. the thinking process that you have followed in buying the particular material”. After explaining the procedure of the game, Mr. Jagat started observing the children. A few minutes later, children came up with their results. The result given by one of the students, Radhika, is given below:

S.No	Material Purchased	Quantity
1	Notebook	20
2	Pen	10
3	Pencil	15
4	Geometry Box	3
5	School Bag	3
6	Textbook	5

Process Followed:

- I have *one* sister and *one* brother. So I thought that they also *need notebooks for one* year. So, I purchased 20 notebooks. I know, we *require more than 20 notebooks*, but **I didn't have money to buy more than 20**. So, I *limited my purchase to 20*.
- The *cost is very low* for pens. So, I thought, **I would buy 10 pens** so that we *three can use them for the entire year*.
- Pencil *doesn't last long*. We *used to buy 4/5 pencils every year*. Also, the *price is very less* for it. So I purchased 15 pencils.
- The *shop sells good quality geometry box and school bag*. So, I thought three pieces of each is enough for us.
- Lastly I purchased text books because **I was left with ? 100 only**. Then **I calculated that, • 80 are required to purchase 5 textbooks**. So, I purchased them. **I didn't have the money** to purchase textbooks for others.

What can we conclude from this activity? Here, the teacher was trying to introduce the chapter on ‘statistics’. He creatively organised an activity and finally used the same data to introduce the concept of frequency. In the Box given above, the letters in bold italic words show childrens’ ways of mathematical thinking. Here, the students confronted a problem that they should buy school materials for themselves and their siblings. Also, they are left with a particular amount. In such a situation, they had used the mathematical thinking to find a solution (here the purchase of materials with the money they have in hand). As a teacher, you may organise innovative activities that help children to develop knowledge in the subject as well as chances to build their mathematical thinking.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

4) What do you mean by mathematisation?

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5) Suggest an activity that would enhance the skill of mathematical thinking of children.

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ENHANCEMENT OF REASONING POWER AND VISUALIZATION

It is reminded that we are discussing the major aims of Mathematics teaching. The next aim is concerned with development reasoning power and generalisation in which a part of the later has been discussed in the previous unit. Reasoning and visualisation go hand in hand and are key processes in validating mathematical knowledge. **Mathematical reasoning enables children to arrive at solutions/judgements/conclusions after manipulating the facts involved in the problems.** To solve problems, children evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how these solutions can be applied. Mathematical reasoning involves two processes, namely: **inductive reasoning and deductive reasoning.** Children knowingly or unknowingly use both inductive and deductive reasoning in various situations. **Inductive reasoning is the process of observing data, recognizing patterns, formulating hypothesis, and making inferences.** It is fundamental to learning of Mathematics. Inductive reasoning, not only develops problem solving skills, but also facilitates learning of Mathematics.

When you draw conclusions on the basis of observations from experience, you use inductive reasoning. Mathematicians use inductive reasoning to find out the true solutions. It proceeds from concrete to abstract, from particular to general and from examples to general formulae. It helps in developing facts, concepts, principles, rules, definitions and generalisations, and basic other elements of Mathematics. In inductive reasoning, we establish a relation by experimenting with numerous examples and wish that it would be the same for similar cases, and hence, it helps to discover new body of knowledge. Mathematics curriculum expects children to develop the skill of reasoning to seek answers for mathematical problems. An activity that can be organised in classrooms to enable children to develop the skill of inductive reasoning is as follows:

Activity 2

The activity is organised to establish a formula to compute the simple interest. Here, the teacher, Mr. Akshay has worked a series of steps to make his children develop the formula of simple interest. The steps are given below:

Step 1: In the first step, Mr. Akshay presented many examples related to the calculation of simple interest (S.I.). The students use one unitary method to calculate S.I.

Example 1: Find the simple interest on ₹1000 at the rate of 4% per annum for 3 years.

Solution

$$\text{S.I. of ₹1000 for 1 year} = ₹40$$

$$\text{S.I. of ₹1 for 1 year} = ₹ \frac{4}{100}$$

$$\text{S.I. of ₹1000 for 1 year} = ₹ \frac{4 \times 1000}{100}$$

$$\text{S.I. of ₹1000 for 3 years} = ₹ \frac{4 \times 1000 \times 3}{100} = ₹120$$

Example 2: Find the S.I. of ₹700 at the rate of 4% per annum for 3 years.

Solution

$$\text{S.I. of ₹700 for 1 year} = ₹28$$

$$\text{S.I. of ₹1 for 1 year} = ₹ \frac{4}{100}$$

$$\text{S.I. of ₹700 for 1 year} = ₹ \frac{4 \times 700}{100}$$

$$\text{S.I. of ₹700 for 3 years} = ₹ \frac{4 \times 700 \times 3}{100} = ₹84$$

Step 2: Here, the teacher asks questions based on the examples worked out above. These are as follows:

Teacher : What is the S.I. in example 1?

Students : ₹120.

Teacher : What is the S.I. in example 2?

Students : ₹84.

Step 3: Next, the teacher asks the students to observe the above examples. Then, he asks the students establish a relation to find the S.I. At this stage, teacher helps children in forming the formula. With the help of the teacher, children developed the formula to find S.I. and are as follows:

$$\text{From example 1, S.I. for 3 years} = \frac{4 \times 1000 \times 3}{100}$$

$$\text{From example 2, S.I. for 3 years} = \frac{4 \times 700 \times 3}{100}$$

$$\text{Therefore, S.I} = \frac{P \times R \times T}{100} \text{ Where; P=Principal, R=Rate and T=Term)$$

Step 4: In the final steps, the teacher provides similar problems and the students are asked to solve them by using the derived formula.

In the above discussion, a formula is derived by looking at a few examples. This way of reasoning is called as inductive reasoning. As a teacher, it is our

duty to provide problems that may result in derivation of facts, concepts and rules of Mathematics. Now, let us discuss deductive reasoning. **Deductive reasoning is exactly opposite to inductive reasoning in which the individual proceeds from general to particular, from abstract to concrete, and from formula to examples.** Generally, deductive reasoning finds application at the higher classes; even then at times secondary teachers utilize it for teaching-learning. Here, the child has to memorise numerous formulas and rules which are used to solve problems. For example, the formula $S.I. = [P \times R \times T]/100$ is used by children to find simple interest. At this stage, children will be given a problem and the formula. The only role of the children is to apply the given data and generate result.

It is hoped that you have understood the concepts of inductive and deductive reasoning. Now, let us discuss, why visualisation is important in Mathematics and how children use it to generate ideas. Listen to a classroom interaction given below:

Teacher Hello students, listen to me. I am giving you a problem. Try to find solution on your own. O.K?

Students Yes sir

Teacher The question is A boy of 96 cm height is walking away from the base of a building at a speed of 1.1m/s. If the height of the building is 5.4m, find the length of boy's shadow after 5 seconds.

In such problems, what will children do first? They will try to make a mental image of the situation. In this case, they will visualise the building and boy. The figure is produced on the paper which would be similar to the following:

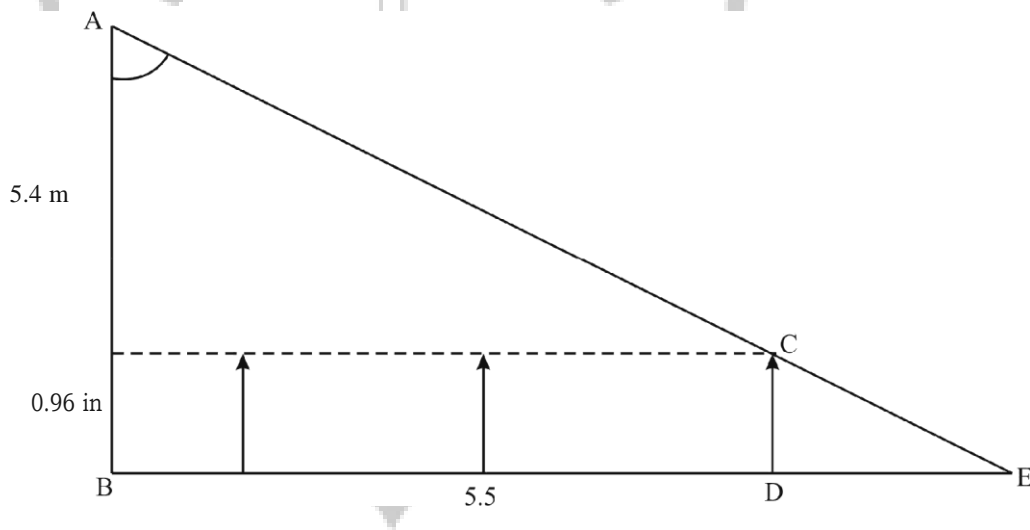


Fig. 2.3

After drawing the figure, the mathematical calculation is tried as follows:

We have to measure the distance $DE = x$ metres

Here $BD = 1.1 \times 5 = 5.5$

In $\triangle ABE$ and $\triangle ACDE$

$\angle B = \angle D$ (Both are 90° as building and boy are standing vertical to ground)

and $\angle A = \angle A$ (Same angle)

so $\triangle ABE \sim \triangle ACDE$ (AA similarity criteria)

Therefore, $BE/DE = AB/CD$

$$\text{i.e. } \frac{5.5 + x}{x} = \frac{5.4}{0.96}$$

$$\text{or } x = 1.18$$

Therefore, the shadow of the boy will be 1.18 m long after walking for 5 seconds. What conclusion do you draw about the classroom interaction? In Mathematics classroom, children do attend problems and their complexity differs. At times, the problems will not be solved directly by applying mathematical equations. Even, children find it difficult to figure out mathematical equation that suits a particular problem. In such cases, children have to develop a mental image of the situation to make sense out of it. The mental image (model) represents the mathematical situation. Even children may visualise like “what will happen if I represent it like ?”. So **visualisation is not just pictures or diagrams, instead, it represents the mental image (model/sketch) of any situation.**

In general terms, visualisation is formation of mental image of something. For example, image of an elephant, school, parliament etc. But in Mathematics, children develop the mental image of mathematical situations and facts. For example, mental image of a cube, bar graph and so on. Why visualisation is important in Mathematics classrooms? How can teachers develop the visualising skills among children? These questions should be considered while designing learning activities. Visualisation helps children understand a complex problem. It helps him to cut down the components to digestible elements so that a connection could be made among the data that appear in the problem. The complex situations can be easily represented as images that help to understand and develop a plan to work out the solution. Generally, visualisations serve three purposes, namely;

- To acts as a support to step into the problem;
- To model the present problem/situation; and
- To plan the strategies for solving the problem.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answer with those given at the end of the Unit.

6) Differentiate between inductive and deductive reasoning.

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DEVELOPING PROBLEM SOLVING SKILLS

In Mathematics classrooms we often find that students are solving problems. Students are able to respond better when they are given direct question where a particular method is to be applied as well as which method is to be applied. However when they are given questions in which they have to decide upon which method to apply, then they become uncomfortable. Most students face difficulty in finding solutions to word problems. This happens in the classroom because students have not developed the skill of problem solving.

Problem solving is a process and comprises of a sequence of steps: **Understand, Think, Try, and Look Back**. Let us discuss these steps of problem solving.

First is to Understand the Problem-Before you can solve a problem you must first understand it. Read and re-read the problem carefully to find what the question is being asked to you to solve. Separate what is given and what is to be found. Write down the information given as points. Write a statement about what is to be found.

Then Think --Once you have understood the problem, look for strategies and tools. Here your previous knowledge will be of great help. So try to remember if you have come across similar situation earlier and recall what you did then. If it turns out to be unfamiliar problem, try to search through your knowledge of Mathematics and identify what could help you in this situation. You can go through your reading material and books. You can work backwards which means look at what is to be found and try to think what can make it possible.

After that Try It—If it is a familiar problem, try if the previous method used works or any modification of the same will work. If it unfamiliar problem try some method which you may have already decided are relevant.

Look Back--Once you've tried it and found an answer, go back to the problem and see if you've really answered the question. Sometimes it's easy to overlook something. If you missed something then check your plan and try the problem again.

As students follow these steps which can be used to arrive at specific strategies. However, selection of the strategy is based on the nature of the problem that they are trying to solve. Some strategies are given below:

- look for a pattern
- draw a diagram or picture
- make a simpler but similar problem
- work backwards
- use a formula
- guess and check
- make and state assumptions
- consider alternative strategies and/or blend strategies
- monitor progress and revise, as necessary.

You as a teacher can help children to select strategy to solve the problem. You can give different types of problems in your class to children where problem solving skill is required. Let see what type of problem you can give to your students.

Example: Raju hired a taxi from his home to the city, which is at a distance of 10 km. During the journey he came to know that taxi charge in the city consists of a fixed charge along with charge for the distance travelled. He paid Rs 105 for the journey. From the city he hired another taxi for travelling up to a hospital, which is 8 km away from the city. He paid Rs 85 to the driver. What are the fixed charges and the charge per km in that city?

Step 1: Understand the problem: Without understanding or comprehending the problem, the student may not be able to solve it correctly. During this process, students may ask themselves some questions like:

- ✓ What is to be found out or shown?
- ✓ What data or information are given?
- ✓ Do I understand all the facts and terms given?
- ✓ Are given data or information sufficient for reaching the solution?, If not, what more information is required?
- ✓ Are there any irrelevant information?
- ✓ How can I restate the problem in a better way?
- ✓ Am I required to draw a picture or diagram?, If yes, then how can I draw it?

Students can pose these types of questions for clearly understanding the problems. All questions may not be applicable to all types of problems. Depending upon the problem, students can frame appropriate questions. In the above example students can ask questions such as:

Question	Expected answer
What is to be found in the problem?	The fixed charge and charge per km for the hired taxi.
What information is given in the problem?	Taxi charge consists of a fixed charge + charge for distance travelled. Taxi charge paid for 10 km travel is Rs 105. Taxi charge paid for 8 km travel is Rs 85.
Are the data given sufficient for arriving at solution?	Yes.

By answering such type of questions students can move to the next step.

Step 2: Then think and Devise a Plan: This step explains the ‘how’ aspect of finding the solution to the problem. Here, students need to think and devise an appropriate strategy for solving the problem. Based on their previous experience, students can come up with a concrete plan. They can initiate this

step on the basis of answers to the earlier queries. If the data given are sufficient, and if students know what to find out, they can link those two informations and can raise another question, “how can I find the desired solution with the help of given data?”

In the above example, students can realise through questioning that, they are expected to find the fixed charge and charge per km for the hired taxi (two variables/unknown).

The data given are taxi charge paid for a distance of 10 km and for a distance of 8 km. This amount consists of fixed charge and charges per km, both are unknown. If these unknowns can be assumed to be x , and y respectively, these statements can be converted into two equations in two unknown, and subsequently, procedure for solving them can be utilised.

Students thinking process can be reproduced as follows:

Taxi fare = Fixed charge + Charge per km \times Distance travelled

For charge paid for 10km travel = 105

i. e, Fixed charge + $10x$ Charge per km = 105

Similarly, fixed charge + $8x$ Charge per km = 85

Assume that, the fixed charge is ‘ x ’ and charge per km is ‘ y ’ the above two can be represented in terms of two equations

$x + 10y = 105$ and $x + 8y = 85$

Based on this analysis, the student can finalise as to which strategy can be used for solving the problem

Step 3: Carry out the plan: After devising a suitable plan for solving the problem, the students can move to execute the plan. At this stage, students have to solve the above mathematical equations in two unknown using any strategy for solving simultaneous equations in two unknown.

They can go for either substitution method or elimination method. Suppose they are using substitution method, they will solve the equations in the following way

$$x + 10y = 105 \dots\dots\dots(1)$$

$$x + 8y = 85 \dots\dots\dots(2)$$

$$\text{From (1), } x = 105 - 10y \dots\dots\dots(3)$$

Substituting the above value of x from (3), in equation (2)

$$\text{We get, } 105 - 10y + 8y = 85$$

$$\text{or } 105 - 2y = 85$$

$$\text{or } -2y = 85 - 105$$

$$\text{or } -2y = -20$$

$$\text{or } y = 10$$

$$\text{From (3), } x = 105 - 10 \times 10$$

$$\text{or } x = 105 - 100$$

$$\text{or } x = 5$$

Therefore, fixed charge is Rs 5, and charge per km is Rs 10.

Step 4: Look back: After finding the solution, it is important to verify that solution. Students may be encouraged to check not only the result, but also the various steps, procedure and calculation also. They may be encouraged to verify the solution by answering the questions like ‘Is the answer reasonable?’, ‘Is there another method of solution that will easily verify the answer?’ , ‘Does the answer fit with given data?’ , etc

For example, in the above problem:

Taxi charge for travelling a distance of 10 km is 105

ie, fixed charge + charge for 10 km = 105

therefore, $5 + 10 \times 10 = 105$

ie, $5 + 100 = 105$

or, $105 = 105$

Similarly, taxi charge for travelling a distance of 8 km is 85.

or, fixed charge + charge for 8 km = 85

or, $5 + 8 \times 10 = 85$

or, $5 + 80 = 85$

or, $85 = 85$

One way to inculcate a positive attitude towards Mathematics and develop interest in the subjects among students is to provide them with various types of problems and help them to solve them independently as far as possible. The satisfaction of the experience of success in solving the problem will definitely enhance their confidence, and in turn, positive attitude towards Mathematics. If we are able to provide problems or tasks that encourage reflection and communication, and those are selected from students’ real life situations, they will take the subject seriously and will appreciate the value of learning Mathematics.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

7) Discuss the steps involved in problem solving with examples.

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8) According to your experience, what are the steps children use to arrive at solutions to mathematical problems?

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DEVELOPMENT OF CRITICAL THINKING IN MATHEMATICS

Critical thinking is defined as meaningful, unbiased decisions or judgments based on the use of interpretation, analysis, evaluation, inferences, and explanations of information as it relates to the evidence applied to a specific discipline. Critical thinking is the way of deciding whether our claim or judgment is true or not. We make reasoned judgments or decisions about what we think or do. So, forming judgments and decisions based on one's own experience or claiming one's own decision true or false with supporting proofs is called critical thinking. A child who thinks critically will himself/herself ask the following questions when he/she faces a perplexing situation/problem (may be a mathematical problem):

- What is the problem? How can I find solution to it?
- Can I solve it with the information I have? If not, what additional information is required?
- Can I solve this problem with the same formula as studied in the classroom?
- If I do it like this, what would be the answer?
- What other strategies and formulas will work here?
- Can this problem be solved by a single method or I should use some other method?

Today, the constructivist approach emphasise 'critical pedagogy' as a learning approach. Whatever the teacher taught is critically analyzed and questioned by the children that ultimately lead to the development of mathematical concepts by their own. As a teacher, how will you develop among children the skill of critical thinking? You may organise activities and let children find the solution by their own efforts. Also present mathematical problems that have multiple ways of solution. For example, teacher can ask the following question:

Question: Find the area of a sector of a circle with radius 5 cm which has an angle 40° . Also, find the area of the corresponding major sector (Use $n = 3.14$).

Solution

Given sector is OPAQ (See figure 2.4)

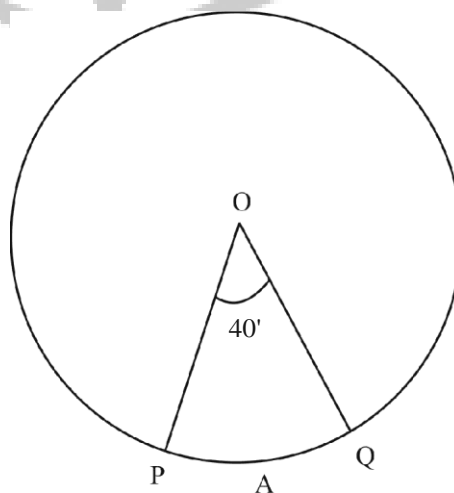


Fig. 2.4

$$\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2 = \frac{40}{360} \times 3.14 \times 5^2 = 8.72 \text{cm}^2$$

$$\begin{aligned} \text{Area of the corresponding major sector} &= \pi r^2 - \text{Area of the sector OPAQ} \\ &= 3.14 \times 5^2 - 8.72 = 69.78 \text{cm}^2 \end{aligned}$$

The example given above suggests the way children will arrive at solution. But, we have an alternate method also. Usually, children do not show interest in attempting alternate methods, and thus, the skill of critical thinking is not developed. But children having the ability of thinking critically, then they experiment with new methods to arrive at solutions. The alternate method that helps to find solution to the above problem is given below:

$$\begin{aligned} \text{Area of the major sector} &= \frac{360 - \theta}{360} \times \pi r^2 = \frac{360 - 40}{360} \times 3.14 \times 5^2 \\ &= 69.78 \text{cm}^2. \end{aligned}$$

In the example given above, the teacher has used a problem that has two ways of finding solution. As a teacher, you may try different methods. Apart from presenting mathematical problems, we can use the following strategies that will help children to develop critical thinking:

- Children may be presented with problems that do not have predetermined solutions and methods leading to answer.
- Provide children challenging questions and problems having multiple strategies to find solution.
- Children may be asked to find real life situations that have application of Mathematics.
- Encourage children to attempt open problems and questions with multiple solutions.
- Make children understand what they are supposed to do about problems.
- Make children focus on the type of problem thus, they are supposed to solve.
- Make children sure that they are working out the problems completely.
- Ask children to identify the mistakes committed while solving problems.
- Motivate children to attempt series of problems of varied types.

Check Your Progress

Note: a) Write your answer in the space given below.

b) Compare your answer with those given at the end of the Unit.

9) Suggest some ways to develop critical thinking among children.

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INTEGRATION OF MATHEMATICS WITH OTHER SUBJECTS

In the previous section, we stated that “Mathematics is the science of all sciences and art of all arts”. Is this statement valid? Why do we make such a statements? Does Mathematics have any relation with science and art? Yes of course! The presence of Mathematics can be seen anywhere including the subjects like sciences, biology, engineering, agriculture, philosophy, psychology, history, geography, drawing, arts, languages and commerce. So, Mathematics has correlation with all other subjects. Today integrated teaching is advocated as it helps children for easy assimilation of subject knowledge. Thus, while organizing learning experiences, teacher should help children to integrate Mathematics and concepts of other subjects. Let us first briefly discuss the correlation of Mathematics with other subjects and then ways of integrating it with other subjects.

Mathematics and Physics: Mathematics is closely associated with physics. In physics, we find many mathematical equations and formulas. For example, $v=u+at$, makes use of the concept of mathematical equation (equal sign). Similarly, the laws of motion, laws of levers, laws of reflection and refraction, laws of electric current, etc. use Mathematics. Light rays, levers, steam engines, telephones and communication devices, electromagnetic rays, electronic and semiconductor devices, etc. have the application of Mathematics.

Mathematics and Chemistry: All chemical combinations are governed by mathematical laws. The formation of a chemical compound is not possible without Mathematics. The basic unit of any substance, atom and the sub particles, of which atom is made up, obey mathematical laws. Chemical equations are controlled by mathematical principles. The huge amount of energy created by an atom is calculated using Mathematics.

Mathematics and Biology: Mathematical principles and facts are applied in all studies concerning botany and zoology. The caloric and nutritional values are calculated using Mathematics. The growth of plants and animals is measured; the respiration and transpiration of water in living bodies etc also use Mathematics. The study of nutrition, growth, maturation, compounds, mixtures, laws of chemical combination, molecular and atomic structure, chemical names and formulas etc all are based on mathematical laws.

Mathematics and Engineering: Mathematics is considered as the foundation of engineering. Surveying, levelling, building construction, construction of electronic and other mechanical devices, dams, bridges etc., use laws and principles of Mathematics.

Mathematics and Agriculture: In agriculture, the money involved, expenditure and income generated are calculated. Similarly, the time to start cultivation of crops and vegetables is analysed. Also, the measurement of plots for cultivation, production per unit area, cost of labour, seed price, etc are calculated using Mathematics.

Mathematics and Social Sciences: Economics employs mathematical principles and languages to interpret social phenomena. The share market operations, country’s revenue statement, budget analysis, etc. use Mathematics.

In geography, the climatic changes, height of mountains, knowledge of rivers, population, moment of winds, area of earth, longitude and latitude, etc. are measured using Mathematics. In history, the historical developments are traced and analysed with Mathematics. Similarly, in philosophy, the basics of all subjects are formed with the help of Mathematics. In commerce, Mathematics is used in accounting and bank-related operations.

Mathematics and Psychology: All the psychological measurements related to human behaviour are collected using appropriate scale constructed using mathematical principles. Also, statistical methods use to organise, analyse and interpret psychological behaviour of any object/subject. Experimental psychology is based on mathematical computations.

Mathematics and Art and Drawing: The various branches of drawing like geometrical drawing, memory drawing, figure drawing, etc. employ Mathematics to produce beautiful colour combinations and pictures. A picture is attractive to eye when the proportion and ratio of colours are perfectly maintained. Even an artist, makes use of his/her mathematical knowledge before attempting to draw a picture on the canvas.

Mathematics and Language: It is language that helps Mathematics to express mathematical equations, laws and principles. Similarly, the medium of expression of any mathematical fact employs the use of language. The funny thing is that, language differs from place to place, but the mathematical idea remains constant irrespective of the language.

Having understood the relation between Mathematics and other subject areas, it is now the task of the teacher to integrate each subject with Mathematics in the teaching-learning process. How is it possible? Let us listen to a conversation. Here, Mr. Ramkishore is teaching the concept of 'arithmetic progression' to his tenth class students. Arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. The fixed number is called the common difference. How will a teacher introduce AP to his/her students? Normally, the teacher may say, "in the series of numbers 2, 4, 6,8,10, etc., each number is 2 more than the previous number. Such a list of numbers is called an AP". The teacher may give a few more examples. The children passively admit the concept and reproduce in the term tests. But, let us take note of the classroom interaction of Mr. Ramkishore.

Ramkishore : Hello children! how are you? Have you done yesterday's home assignment? Let me check it.

Students : Yes sir.

For a while, Ramkishore gets engaged in evaluating the home assignments. After checking the home assignments, he continued:

Ramkishore : Let me tell you an incident. The previous day, I went to a shop that sells pesticides. Do you know about pesticides?

Some students said 'yes' while a major group were unaware of pesticides. Then, teacher continued:

Ramkishore : I will tell you. Pesticides are substances which are used to destroy pests.

Ramkishore continued to talk about pesticides. He explained different kinds of pesticides, where they are found, the techniques to use pesticides, and so on. Then, he started narrating the incident;

Ramkishore : So, where did I stop? Aah.. yes. yes. I went to the shop. Then for ten minutes I watched the people buying pesticides. Then what I found was that a man purchased 1 kg pesticides, then the other man purchased 2 kg (he may be having more cultivation), a third man 3kg, fourth man, 4kg and so on. Now, my question is can you tell, the quantity of pesticides purchased in order?

Students : It is very easy sir. 1 kg, 2kg, 3kg, 4 kg, 5kg.....

Ramkishore : let me repeat, 1,2,3,4, 5, isn't it?(he write the numbers on the board)

Students : Yes Sir.

Ramkishore : Now, my next question. Do you find any relation in the numbers given above?

The discussion prolonged till the conclusion, Mr. Ramkishore introduced the concept of AP. In this example, Mr. Ramkishore has tried to correlate Mathematics with the subject of agriculture. In the process, he discussed the different aspects of the concept 'pesticide', a topic of agriculture. In a similar way, Mathematics can be correlated with other subjects.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

10) Why is integration important in Mathematics teaching?

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11) How will you teach Mathematics by integrating it within concepts biology?

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LET US SUM UP

Aims are broad in nature while objectives are the means to achieve it. Mathematics teaching enables to develop among children objectives like critical thinking, ability to think in mathematical terms, skill of problem solving, development of reasoning power and visualisation, etc. In this unit, we

have extensively discussed the broad aims and objectives of Mathematics teaching at secondary level with adequate examples. Thereafter, the relation of Mathematics with daily life experiences has been discussed citing examples. As teacher, you may organise learning experiences by connecting with daily experiences of children in and out of the school context. At the end of the Unit, the correlation of Mathematics with other subjects has been discussed which would enable you to plan learning activities integrating concepts belonging to other subjects. Integrated teaching facilitates children to develop mathematical concepts in a much better way.

UNIT END EXERCISES

- 1) Organise a discussion in your class on the topic 'enhancement of reasoning power of children'. Prepare a report.
- 2) Suggest a few strategies that can be organised in the classroom to promote the problem solving skills of children.
- 3) Ask children of your class to identify the mathematical ideas involved in their day to day activities
- 4) Identify a mathematical concept and explore the possibilities of teaching the same by integrating with other subjects.

ANSWERS TO CHECK YOUR PROGRESS

- 1) Refer section 2.3.1.
- 2) Following are the general objectives of teaching mathematics
 - Develop interest and positive attitude towards Mathematics.
 - Apply mathematical knowledge to solve real life problems.
 - Develop the skill to use algorithms in problems solving.
- 3) The students will be able to:
 - Recall the definition of probability.
 - State the formula for probability.
 - Cite examples from daily life involving probability.
 - Solve problem related to probability.
- 4) Mathematisation refers to the act of interpreting or expressing mathematically, or the state of being considered or explained mathematically.
- 5) Role play may be organised. Children may be assigned different roles such as sales man, cashier, etc. Let them be involved in activities that occur in a textile showroom.
- 6) Refer section 2.4
- 7) Selection and formation of problem, Presentation of the problem, Formulation of hypothesis, Collection of relevant data, Analysis and organization of data, Formulation of a tentative solution, Drawing conclusion
- 8) Do it yourself.

- 9) Provide challenging mathematical problems; ask them to complete projects, etc.
- 10) Refer section 2.8
- 11) Refer section 2.8

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