ASSUMPTIONS, POSTULATES, AXIOMS AND HYPOTHESIS IN MATHEMATICS

The concept of Assumptions, Postulates and Axioms was given by Euclid who was Greek mathematician. He is often referred as 'Father of Geometry'. It was known that he taught mathematics about 300 BC in Royal School at Alexandria in Egypt that has been founded by Ptlomey the successor of Alexander the Great Euclid must have studied in Plato's academy in Athens and learnt the geometry of Eudoxus and Theaetetus of which he was familiar. As a teacher he was probably one of the mentors of Archimedes. He was best known for his treatise on mathematics, 'Elements' was born about 325 BC.

THE ELEMENTS

Euclid's Elements has been a driving factor in the development of mathematics and mathematical logic for twenty three centuries. "The Elements' has been used for centuries in Western Europe as a geometry text-book. The Elements' contains 13 books, each begins with definitions, postulates and common opinions, then proceeds to obtain results by rigorous geometrical proofs. Euclid assumed certain properties, which were not to be proved. These are called assumptions. These assumptions are actually 'Obvious Universal Truths'. He further divided them In two types: Axioms and Postulates.

ASSUMPTIONS

Assumption is the act of taking something for granted. It is presumption or an assuming of power or possession of something.

1. Assumption is a statement that is assumed to be true and from which a conclusion can be drawn. It's a hypothesis that is taken for granted.
2. An assumption is a supposition or something that is taken for granted without questioning or proof. What we believe is true without proof.
3. Assumptions need not be true.
4. Assumptions gives birth to axioms and postulates.
5. When doing any sort of maths word problems, we usually have to make few assumptions in our calculations. Otherwise a question may be too complicated to solve. E.g. On the assumption that he has been injured, We can infer that he will not play. e.g. If we are asked to find the area of rectangular garden plot that is 6 feet by 4 feet, we have to assume that:
   (i)   The plot is perfectly rectangular (even though in reality one of the corners may be off by a couple of degrees).
   (ii)   The measurements are exact (even though in reality its easy to misread a tape measure by $1/16^{th}$ of an inch)

   This way we can do the calculation and get 24 square feet.

6. Assumption is basic to an argument.
7. In assumption we tests the validity or effect of something e.g. Every natural number has a successor that is itself a natural number.

AXIOM

An Axiom is a statement that is considered to be true, based on logic; however it cannot be proven or demonstrated because it is simply considered as self evident. Basically, anything declared to be true and accepted, but does not have any proof or has some practical way of proving it, is an axiom.m The word Axiom comes from the Greek Axióma a means worthy or fit or that which commends itself as evident. These are the strong self evident truths.

1. Axiom is a universally accepted principle or rule.
2. It's a preposition in logic or mathematics that is assumed without proof for the sake of studying consequences that follow from it.
3. An Axiom is a statement assumed to be true for the sake of expressing a logical sequence. They are the principal building blocks of proving statements Axioms, serve as the starting point of other mathematical statements.

The set of Axioms of a mathematical system has following **characteristics:**

1. Axioms are propositions which are assumed and accepted without any evidence to prove it.
2. Axioms are consistent.
3. Axioms are not superfluous.
4. Axioms are adequate.
5. Axioms are considered as "Rules of Game".
6. Axioms are analytic propositions.
7. It serves as a starting point from which other statements are logically derived.

The Axioms are acceptable in the sense that there is no evidence to the contrary; on the other hand there are enough evidences to show that axioms are true.

Axioms can be categorized as **logical or non-logical,**

Logical Axioms are universally accepted and valid statements, while non-logical axioms are usually logical expression used in building mathematical theories.

POSTULATES

The term "Postulate" is derived from the Latin "Postular", It's a verb which means "to demand". The master demanded his pupils that they argue to certain statements. Unlike axioms, postulates aim to capture what is special about a particular structure. "It is possible to draw a straight line from any point to any other point", "It is possible to produce a finite straight continuously in a straight line", and "It is possible to describe a circle with any center and any radius" are few examples for postulates illustrated by Euclid.

Postulates given by Euclid are:

Postulate 1: A straight line may be drawn from any one point to any other point.

This postulate tells us that at least one straight line passes through two distinct points, but it does not say that there cannot be more than one such line. So, Euclid has assumed that there is a unique line joining two distinct points. We state this result in the form of an axiom as follows: Axiom: Given two distinct points, there is a unique line that passes through them. Only one line PQ passing through P and Q.

Postulate 2: A terminated line can be produced indefinitely.

Euclid called terminated line, now-a-days we call it line segment. So, second postulate states that a line segment can be extended on either side to form a line.

Postulate 3: A circle can be drawn with any centre and any radius.

Postulate 4: All right angles are equal to one another.

Postulate 5: If a straight line falling on two straight lines make the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles. E.g. Line PQ falls on line AB and CD such that sum of the interior angles 1 and 2 is less than 180° on the left side PQ. Therefore, the line AB and CD will eventually intersect on the left side of PQ.

The first three postulates are postulates of construction. The fourth postulate states that all right angles are equal. The famous fifth, or parallel postulate states that one and only one line can be drawn through a point parallel to given line.

Now-a-days, 'Postulates' and 'axioms' are terms that are used interchangeably and in the same sense. Postulate' is actually a verb when we say "Let us postulate" we mean, "Let us make some statement based on the observed phenomenon in the universe." Its truth/validity is checked afterwards. If it is true, then it is accepted as a 'Postulate'.

So, postulates are synthetic. It is based on some observed phenomenon in the universe. Postulates are applicable only to specific science being considered.

HYPOTHESIS IN MATHEMATICS

The word hypothesis is composed of two words 'Hypo' and 'Thesis'. Hypo literally means under or below and Thesis means of reasoned theory or rational viewpoint. Accordingly, hypothesis would mean a theory which is not fully reasoned. In other ways, hypothesis is a theory entertained in order to study the facts and examine the validity of the theory. Thus hypothesis is a tentative statement about the solution of a problem. It offers a solution of the problem that is to be verified empirically and based on some rationale.

Operational Meaning of Hypothesis

'Hypo' means composition of two or more variables which is to be verified. "Thesis' means position of these variables in the specific frame of reference. Hypothesis is called a leap into the dark. It is a brilliant guess about the solution of a problem.

Definitions

"A hypothesis is defined as a suggested answer to a problem."-J.C. Town

"A hypothesis is a conjectural statement of the relation between two or more variables."-FN. Kerlinger

Examples of Hypothesis in Mathematics

Hypothesis is the part of a conditional statement just after the word 'If'.

Example 1:"If all the four sides of a quadrilateral measure the same, then the quadrilateral is a square." A lateral

In the above example, the hypothesis is "All the four sides of a quadrilateral measure the same."

Infact this hypothesis is sometimes true, its true only for a square and a rhombus, not for other quadrilaterals like rectangle, parallelogram, trapezium etc

Example 2: Basic Theorem in Calculus

"If the function f is continuous on closed interval [a, b] then it is Riemann Integrable over the interval

[a, b]. The hypothesis of this theorem is "f is continuous on closed interval [a, b]."

The conclusion is "f is Riemann Integrable over [a, b]."

Nature of Hypothesis

1. A good hypothesis is clear, precisely stated and testable in some way.
2. Creation of clear hypothesis requires clear general mathematical thinking.
3. Each hypothesis may lead to formulate another hypothesis.
4. A hypothesis is a verbal statement.
5. It is a condensed generalization.
6. The hypothesis is based on some earlier theory and some rationale where as postulates are taken as granted true.
7. An assumption is the assumed solution of a major problem, it may be partially true.
8. It indicates the tentative relationship between two or more variables.
9. It guides the thinking process.

Conclusion

Hypothesis serves as a powerful beacon that lights the way for any research or problem. It gives help in deciding the direction in which one has to proceed.